

Data Modeling Supports the Development of Statistical Reasoning

Data modeling is an approach to learning and teaching statistics that engages students in ways of thinking about variability that approximate how professionals in the discipline of statistics think about variability. Traditionally, data and statistics have been taught apart from chance and probability. Data Modeling integrates these two strands, so that learning about data and statistics helps students think about chance, and learning about chance helps a student interpret statistics and data. The two strands come together as students invent and revise models that explain variability in data that they have generated. Like professionals in data analysis, students invent and adapt representations, measures and models of variable data.

The Data Modeling curriculum is a story. It is the combined account of the unique histories of students and their teachers from many districts engaging with fundamental ideas of statistics and probability over a series of designed events that build on student understanding. The story is based on these essential themes:

- Data Modeling creates opportunities for students to construct data by engaging in simple processes that generate variability, such as repeated measurement of the same attribute or production of a product. By participating in these processes, students link process to variability.
- Students invent visual displays, measures and models of the data generated by these variability-producing processes.
- During small-group and whole-class conversations, student inventions are compared and contrasted with an eye toward the mathematical ideas that guided their creation.
- Conventions, such as forms of display used to visualize data or statistics used to measure characteristics of a distribution, are solutions to problems of visualizing, summarizing, and modeling the variability inherent in chance processes. Student inventions help make the rationale for these mathematical conventions more accessible and visible. Conventions typically solve problems that students encounter as they invent.
- As students progress in the curricular sequence, initial understandings of visualization, measures and models of data are extended and elaborated to explain new contexts of variability. Once initially learned, core concepts are re-used and re-contextualized, never abandoned.
- Construct maps describe typical progressions of student thinking about core concepts of visualization, statistics, chance, and

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modeling. These constructs provide the foundations for formative (during instruction) and summative (after instruction) assessments, so that assessments are geared toward inferring states of student knowledge from student responses, rather than simply focusing on “right” or “wrong.”

- Teachers use student responses to formative assessments to plan and conduct formative assessment learning conversations. During these whole-group conversations, selected student responses are compared and contrasted to emphasize mathematical ways of thinking, so that all students have opportunities to revise and refine their thinking.

These themes underlie the type and order of investigations students encounter during 7-8 weeks of instruction. The investigations are organized into units that position students to participate in ways of acting and thinking that are characteristic of how statisticians act and think.

A snapshot of each of the units provides a brief introduction to the curriculum, but if we refer to unfamiliar concepts in these snapshots, don't worry. We developed Data Modeling in partnership with teachers, and teachers now lead our professional development. They will be available to help you learn about these ideas and to help you develop them with your students. Each unit contains a section, entitled Mathematical Background, which highlights the mathematical concepts and procedures that are the focus of that unit. Other supporting materials, including classroom video illuminations of the expected evolution of student thinking described by each construct map and teacher-developed supplements to the units, are found at *modelingdata.org*.

Unit Summaries

Unit 1, *Inventing Displays*, engages students in a simple process: Measure the length of an object, such as the arm-span of a person or the perimeter of a table. Each student independently measures the same attribute. The tool provided for measurement, a 15 cm ruler, ensures that students' measurements will be different due to chance variation in how they measure. The aim is to help students understand that variability is the result of a repeated process. Here each person's measurement of the length is a repetition of the process of measuring, which includes moving the ruler. If time allows, students measure the same attribute again with a meter stick or tape measure, and these additional measurements are set aside for future use. Measuring with a meter stick or tape measure provides students with an opportunity to relate a change in process with a change in the resulting variability of the measurements. Students create displays of the ruler data to illustrate "something they noticed" about the collection of data from the entire class. Students then compare the various displays that they make for aspects of the data that are revealed and concealed by each data display. The shape of the data visible in the display results from designers' decisions about how to structure the data. Important mathematical means for structuring data include ordering, counting, grouping, and using the measurement scale of the batch of measurements. Milestones of student progress are illustrated by the Data Display (DaD) and Meta-Representational Competency (MRC) constructs.

Unit 2, *Inventing Center*, challenges students to create a method, a series of steps that anyone could follow, to estimate the "true" or real measure of the length. The displays generated during the previous unit guide students' invented methods. Students compare their methods with those of others, focusing on what aspects of the data the inventor employed to generate the measure. The big idea is that a statistic—the result of applying the steps of the invented method—measures a characteristic of a distribution. Here that characteristic is the central tendency of the collection of measurements. Students compare their methods with traditional measures of center: mean, median, and mode. Typically, students' inventions include the median and the mode. Further investigations in the unit explore properties of the mean as a fair share and the median as the point that splits the data into two equal-number shares (50%). A teacher supplement describes an investigation of the mean as a balance point. Progress in student conceptions of statistics is illustrated by the Conceptions of Statistics construct.

Unit 3, *Inventing Precision*, employs what students have learned from Units 1 and 2 to focus on the variability of their data. Students invent a measure of precision. Precision refers to the tendency of the measurements to agree, to be consistent from measurer to measurer. Students use a software tool, TinkerPlots, to develop this measure (a statistic of variability). Some student inventions use the distance between each measurement and the center, that is, a deviation of each value from the median or mean, to summarize the tendency for the measurements to agree. This solution is an analog to the mean deviation statistic. Other inventions tend to focus on the boundaries of the center clump, an analog to the inter-quartile range. Yet other inventions focus on the extreme scores—the range statistic. Most student solutions are unconventional, but contain the seeds of the mathematical thinking that motivates more conventional approaches. Once again, students compare their methods with those of others, focusing on what aspects of the data the inventor employed to generate the measure. With teacher guidance, the class considers the quality of the measure proposed: Does it provide a way to describe the tendency of the measurements to agree? What happens to the statistic if some of the measurements change? Students' invented statistics are compared to three conventional statistics: range, mean absolute deviation, and inter-quartile range. Often, teachers ask students to compare the precision of measure obtained with the 15 cm. ruler vs. the meter stick tools in Unit 1.

Unit 4, *Exploring Generalization*, amplifies the theme of mathematical generalization: How do statistics behave when new samples are considered? If teachers have not already done so, one investigation considers the effects of measuring a length, such as an arm span, with a more precise tool, such as a meter stick or tape measure. Measuring an arm span with a meter stick typically does not substantially alter the center of the distribution, but it does reduce the variability of the measurements. Do the statistics of center and variability (precision) capture what stays the same and what changes when the measurement process is altered by the use of the better tool? Unit 4 introduces new contexts of production. In Toothpick Factory, students produce packages of toothpicks and use center statistics to infer the target value (i.e., how many toothpicks per package). Statistics of variability measure the consistency of production (the tendency to pack the same number of toothpicks in each package). Students use these statistics to compare two different methods of producing toothpick packages. In Rate Walk, students walk 10 meters at different target rates (0.5 m/sec, 1 m/sec, and 5 m/sec.). They use statistics of center and variability to decide at which rate they performed best. Other

production process contexts are available on the website, including [Candy Factory](#) and [Fantasy Football](#).

Unit 5, *Investigating Chance*, brings together all of the concepts from Units 1-4 to investigate chance. Students work with hand-held spinners and with TinkerPlots to explore the behavior of 2-color spinners. The purpose is to develop understanding of probability as a measure of uncertainty. The key idea is that the outcome of any single repetition cannot be known in advance, but with many repetitions, the uncertainty of particular outcomes can be measured. Students examine two complementary approaches to estimating probability, theoretical and empirical. They explore the probability of joint, independent outcomes, such as the sums of the outcomes of two spinners, from these perspectives. The unit includes explorations of empirical approximations to the sampling distribution of one or more statistics, in light of changes in number of repetitions of a process (the sample size is the number of repetitions). The sampling distribution is the cornerstone of statistical inference. The [Chance](#) (Cha) and [Conceptions of Statistics](#) (CoS) constructs describe benchmarks of conceptual progress about probability and about the sampling distribution of a statistic.

Unit 6, *Modeling Measurements*, encourages students to integrate data and chance by constructing and revising models that explain the variability of their measurements or of their productions (e.g., counts of the number of toothpicks in each package). Students build and test models of the process of measurement that resulted in the arm span (or some other attribute) measurements they previously obtained. Their models of the measurement process estimate both the true measure of the attribute and random errors made as they measured. For example, just by chance, gaps and overlaps in measurement occur due to difficulties in using the short ruler to measure a long distance, such as an arm-span. Students identify sources of error like these and construct chance devices to model each source. Then, they combine the devices that model chance errors with the estimate of the true measure to produce a model of the measurement process. They compare their models' outputs to their sample data and explore the models' behavior from sample-to-sample, using the concept of sampling distribution to judge the adequacy of their models. They even build and test a "bad" model. The [Modeling Variability](#) (MoV) construct illustrates students' conceptions of modeling.

Unit 7, *Making Inferences in Light of Uncertainty*, employs modeling and sampling distributions based on these models, to make evidence-based inferences about claims regarding different samples of data. For example,

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in light of a model of the measurement process of a person's arm-span, how likely is a particular sample mean arm-span to represent the process of measuring that particular person? The unit also instigates a shift to a context of natural variability. Students participate in an eye illusion experiment and use statistics and models to justify claims about whether or not the illusion affected their perception. The Informal Inference (InI) construct describes milestones in students' reasoning about statistical inference.

As you and your students engage with the problem events in Data Modeling, you add to the narrative, writing yourselves into our developing narrative about probability and chance. Welcome to the story!

Formative Assessments

Each unit includes a formative assessment and a scoring rubric that helps teachers relate student responses to levels of one, and sometimes, two constructs. The rubric guides selection of student responses for further class discussion—we call these discussions assessment conversations. The aim of the assessment conversation is to bring out different ways of student thinking about a statistical idea and to help students make progress toward more powerful ways of thinking. More powerful ways of thinking are those that help students reason more generally about a variety of situations in which data and chance play important roles. Instruction is often extended and modified in light of student responses to formative assessments. For example, teachers often modify a formative assessment item slightly and ask students to use what they have learned from the conversation to respond again to the item.

Classroom Community

Using Data Modeling as a tool for learning depends on the prior existence or concurrent development of classroom discussion and teacher questioning directed toward student thinking. For our purposes, classroom discussion is a conversation among students and between students and teacher about important mathematical ideas. It is a dialogue among the students and teacher as they attempt to make meaning together via an exchange of ideas. This view of discussion is often at odds with what most teachers mean by discussion, in which the teacher asks a question, calls on a student, the student answers, teacher evaluates the response (right/wrong), then asks another question. This pattern is sometimes referred to as IRE (initiate-respond-evaluate) cycle and is a traditional teacher-questioning model, because only the teacher asks the questions and evaluates the responses. In our view of classroom discussion, students' talk dominates, as they build ideas with each other. Students are likely to ask questions of themselves and each other and to share ideas. To emphasize the contrast, we often use the term "kid talk," rather than classroom discussion, because that's the part that is most important. It tells us what the children do and don't understand. Teachers still ask questions, but those questions are designed to elicit and elaborate on student ideas..

In order for students to share their ideas and risk "being wrong" in public, classroom norms that define, actualize, and constrain student talk must be in place, in both large- and small-group work. Examples of helpful constraints include no "dissing" by word or gesture, and we discuss ideas, not people. The teacher plays an important role in modeling, reminding, and enforcing these norms so that students feel safe to talk. Teacher questioning provokes and guides classroom discussion. Teachers formulate queries that prompt, clarify, amplify/enhance, and extend student thinking about the Data Modeling concepts. The Data Modeling curriculum units each include many examples of teacher questions, when to ask them, and likely responses from students. In addition, to guide culminating conversations about important mathematical ideas, Talk Moves for each unit are available on the web site (datamodeling.org). Talk Moves are a core of questions that teachers can use to start a conversation, keep it going, and ensure that at its end, students have had the opportunity to learn about important mathematical ideas. An excellent resource for learning more about classroom discussions and teacher talk is:

Chapin, S., O'Connor, C. and Anderson, N. 2009. *Classroom discussions: Using math talk to help students learn, 2nd edition*. Sausalito, CA: Math Solutions.

To support sharing of ideas, students should keep a written record, a learning journal, of their thinking to which they can refer during discussions. This record may include words, numbers, graphics, and/or symbols. Students use the record to document their ideas, interpret observations, develop arguments, and provide evidence for their thinking about concepts they encounter. As students progress through the units, the record allows them to revisit and reconsider their changing ideas. It also allows the teacher to review the development of students' thinking and make appropriate adjustments to instruction in response. Keeping all of their thinking in a math notebook simplifies storage and retrieval of students' work.

Academic Language

Exchanging ideas in the math classroom obliges teachers to model, and students to engage with, the academic language of the discipline. Educators have recently heard a great deal about Academic Language and its importance to student success in school. According to Goldenberg (2008, p. 9), academic language “refers to more abstract, complex, and challenging language that will eventually permit you to participate successfully in mainstream classroom instruction. (It) involves such things as...possessing and using content-specific vocabulary and modes of expression in different academic disciplines such as mathematics and social studies.” Academic language is much more than a list of pertinent vocabulary; it is, instead, how knowledgeable people communicate in a particular field of study.

In classroom discussions, students should receive multiple opportunities to try out both the generic and content-specific components of academic language in mathematics. By *generic*, we mean school words that are used across subject areas, such as compare, define, and synthesize. Content-specific language refers to words and phrases that help people in disciplines communicate and share ideas.

To support the development of academic language, teachers may create a Word Wall of relevant terms that allows students to consult the list whenever necessary in the communication process. However, the terms are developed with students on an as-needed basis and recorded on the Word Wall only after students have constructed the ideas to which they refer. In other words, we do not advocate pre-teaching vocabulary. Students first encounter concepts within activity structures. After engaging with, talking about, and clarifying their understanding of the idea, the teacher may introduce the appropriate term, telling students that mathematicians agree to call this ____.

A second support of academic language is a posted chart of often-used phrases, summary statements, and argument structures that help children engage in the social practices and conventions of mathematics. These may include: I noticed that ____; I agree/disagree with ____ because ____; I used to think ____, but now I think ____; I learned that ____; I know that ____ because ____; I conclude that ____; In summary, I think ____; among many others. If teachers model these structures and explicitly teach and repeatedly encourage children to use them in conversation they will become the norm for students during listening, speaking, and

recording/representing. As with the Word Wall, students contribute to this resource and then rely upon it.

A third support of academic language is the math notebook. Here students can record descriptions and graphics, try out ideas, notice patterns, and build arguments with the evidence they have compiled. They can revisit their thinking to see both its origin and its progress. Teachers can review the notebooks to gauge student understanding and revise teaching in response.

Finally, establishing a question wall that records student-generated questions invites students to participate in the practice of mathematics because their classroom tasks become investigations into the novel using the familiar. This is what mathematicians do.

Goldenberg, C. (2008) Teaching English language learners: What the research does—and does not—say. *American Educator*, Summer (pp. 7-44).

Academic Language by Unit

Below are tables of Academic Language for each Data Modeling Unit. Some terms that maintain their common meanings (collection, measure, lightweight) are also included for consideration with English Language Learners. Others are included for students who may be unfamiliar with a context (batting average, fouled). Teachers need to be aware that not all children will understand the meaning of all of the words used.

With this list, teachers will know the context of terms (where they were introduced and their frame of reference). In addition, only the first reference to the term is recorded. It is our experience that teachers tend to use the language of the discipline within the classroom and eventually many students adopt it. However, teachers will wish to name concepts that have been developed so that eventually everyone is using conventional terms and referents (what the term means). By Unit 5, the terms are used in conjunction with each other to form phrases representing concepts. For example, sample *variability* is introduced in Unit 3, explored more thoroughly in Unit 4 and extended in Unit 5 to include *sample-to-sample variability* to form yet another concept. Terms are also combined to form definitions or describe processes and conditions, and definitions are combined to explain other concepts. Therefore, students will be expected

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to understand and utilize most of the terms, phrases, and structures below to name learned concepts.

Unit 1 Student Language Demands

Arm span	Measure(ment)	Relate(d, ionship)	Real Length
Possible value	Over-estimate(ors)	Under-estimate(ors)	Display
Trend	Data set	Shape (of the data)	Notice
Values	Visible	Order	Groups of Values (or Bins)
Scale	Graph	Maximum Measure	Gaps/Laps
Overlaps	Holes	Process	Chance
Sample	Performance	In general	Lightweight
Argument	Difference(s)	Information	Unusual
Frequency/Count	Dot/Line Plot	Center clump	Histogram

Unit 2 Student Language Demands

Best Guess	True Length	Consider	Invent
Method	Determine(ing)	Statistic	Center
Characteristic(s)	Attend to	Estimate	Real Value
Main Idea	Repeat(able)	Distribution	Focus
Actual	Situation	Intend(s)	Propose(s)
Mean	Median	Mode	
Represent	Central Tendency	Equivalent	Remain(ing)
Fair-share(d)	Sum/N	Total	Circumference
Exercise	Stroke(s)	Mid-range	Outlier/Extreme Value
Difference Score	Balance	Sample Size (how many cases)	Repeated values

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Unit 3 Student Language Demands

Identical	Exact	Tend(ency)	Alike
Precise(ion)	Partition	Measure of Precision	Agree(ment)
Calculate(ing)	Consistent(cy)	Describe	Patterns
Variability	Spread	Percentile	Result
Compare	Mean absolute deviation	Range	Interquartile range (IQR)
Quartile	Percent	Distance	Deviation
Mid-50	Range	Divider Tool	Hat Plot

Unit 4 Student Language Demands

Improve(d)	Attribute	Collection	Sketch
Particular	Organize(ing)	Allow	Evidence
Prediction	Sensible	Crude (tool)	Contribute(d)
Result(s)	Target	Rate	Line Method
Split Method	Meter(s)	Accurate	Inaccurate
Decision	Inter-quartile range	IQR	Generate
Vary	Variability	Production/Manufacturing	Mean Absolute Deviation

Unit 5 Student Language Demands

Expect(ation)	Outcome	Probability	Event
Ratio	Proportion	Trial	Sample
Represent(s)	Infinite	Population	Combine(d)
Influence(d)	Extreme	Visual	Spinner
Strategy	Two-region	Equal Probability	Percentage
Area	Observe(ation)	Case	Sampling Distribution
Central Tendency	Median of sampling Distribution	Central tendency of the sampling distribution	Sample-to-Sample
Repeat/repetition	Compound Event	Possible Outcome(s)	Impossible Outcome
Sum(s)	Sample Space	Likelihood	Account (for)
Strike	Record	Attached	Construct
Plot	Frequency	Frequency Plot	Challenge
Equal area	Sector	Obtain(ed)	Fouled
Free-throw	Attempting	Batting average	Model
Percent	Carnival	Slot Machine	Window
Ratio of target outcomes to all possible outcomes = Probability, a measure of chance			

Unit 6 Student Language Demands

Random/chance error	Model parameters	Model run	Model fit
Signed arithmetic	Magnitude	Skew(ed)	Correspondence
Statistical inference	Componential model	Model parameters	Components
Fluctuations	Analysis of variance	Inadvertent	Symmetric
Significant	Simulation	Realistic	Ideal
Average/median deviation	Perceive	Elicit	Provoke
Yield	Identical	Iteration	Enact
Tactic	Differentiate	Commonplace	Misreading
Approximation	Revise	Manufacturing	

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Unit 7 Student Language Demands

Open-arrow	Closed-arrow	Bootstrapping	Normal
Exchange	Simulate(d, ion)	Engineer	Diameter
Sharp-eyed	Approximate (ion)	Typical	Perfection
Slack	Statistical inference	Decision	Threshold
Conclusion	Convention(s, al, ally)	Cautious	Line segment
Congruent	Endpoint(s)	Illusion	Visual illusion
Exchange	Interpret	Condition	Replacement
Batter	Ballpark	Professional	Skilled
Nonrandom	Influence	Inspect	Mixer
Counter	Remaining	Sampler	Modify
Symmetry(ic)	Evaluate	Claim	Usual/unusual
County	Birth rate	According	Consider(ing)
Resampling	Increase	Contract	Fertilize
Unfertilized	Combine(ing)	Moisture	Weight
Organic	Assume	Fast Plant	